Predicting the Electrical Equivalent of Piezoceramic Transducers for Small Acoustic Transmitters

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Abstract—**Tubular lead zirconate/lead titanate ceramics are a popular choice for small and medium size ultrasonic projectors. The elements of the simplified electrical equivalent circuit are discussed, and a method for determining element values at or near resonance is described. Results are presented for typical cylinders from 6.3 mm to 76 mm in diameter. An example illustrates the application of the method in the design of an efficient output stage for an oceanographic telemetry transmitter.**

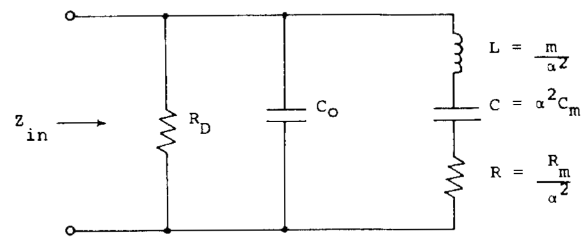

Fig. 1. Electrical equivalent circuit.

I. Introduction

TUBULAR LEAD zirconate/lead titanate piezoelectric ceramics are perhaps the most popular projectors for small and medium size ultrasonic telemetry transmitters. The shape is ideal for use with most batteries, radiation is nearly omnidirectional, and they possess great inherent strength. High electric-to-acoustic efficiency is essential to long life transmitters and is only achieved if the mounted piezoelectric transducer can be accurately characterized. Knowledge of the equivalent electric circuit for the transducer enables the design of an appropriate power amplifier and the prediction of transmitter performance.

This paper describes a procedure for determining the component values of the equivalent circuit of a thin wall ceramic cylinder used in the radial mode of vibration. This procedure, while applicable only at or near resonance, eliminates the requirement for a test tank and expensive electronic instrumentation often used to measure the impedance of a sample transducer. The necessary data for this procedure can be obtained from the manufacturer's literature.

II. Transducer Equivalent Circuit

Equivalent circuit techniques have been employed for many years to obtain solutions to electrical and mechanical problems. Application of these techniques to piezoelectric elements has been well treated by Mason [1], [2] and others. For the purpose of this discussion, the simplified equivalent circuit of Fig. 1 is employed.

The circuit consists of an electrical storage element and the equivalent of a mechanical oscillator. The resistance R_D represents losses in the dielectric, while C_0 is the "blocked" capacitance, or that which would be measured if the transducer could be prevented from vibrating. The components of the mechanical arm consist of the vibratory mass and stiffness and the acoustical impedance of the medium acting on the surface area of the cylinder. If losses within the transducer element and its mounting are assumed negligible (a reasonable approximation in a large number of cases), then the R of the mechanical arm is due solely to the loading imposed by the medium [3]. In the simple case when the transducer is large compared with the wavelength of acoustic radiation, plane waves will be propagated, and the specific acoustic impedance of the medium is real and equal to $\rho_0 c$, which is 1.54×10^6 kg/m$^2 \cdot$ s for sea water and 1.48×10^6 kg/m$^2 \cdot$ s for fresh water. Thus, the motional resistance R_m is given by the expression

$$R_m \cong R_r = \rho_0 c A \tag{1}$$

in which A is the radiating area of the transducer. The quantities m and C_m are the effective mass and compliance, respectively.

An important limitation of this circuit is that, in the actual mechanical system, stiffness and mass are distributed, but the equivalent C and L are lumped. The result is that, while the electrical circuit has only one resonant frequency, the mechanical system can resonate at higher harmonics. A more detailed circuit which incorporates the effects of harmonic resonances, as well as support structures, is discussed by Hafner [4].

The transformation ratio α relates the force generated in the blocked transducer to the generating voltage and is often expressed as

$$\alpha^2 = Z_{\text{mech}}/Z_{\text{elect}}. \tag{2}$$

It is this ratio which enables the designer to construct the electrical equivalent of the loaded, vibrating transducer. The transformation ratio is now developed using the expressions for the response of a transducer undergoing forced harmonic vibration and for the response of a series

Manuscript received March 7, 1984.
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0018-9537/85/0100-0061$01.00

electrical circuit excited by a sinusoidal voltage [3]:

$$\omega F \cos \omega t = m \frac{d^2u}{dt^2} + R_m u + \frac{1}{C_m} \int u \, dt \tag{3}$$

$$\omega V \cos \omega t = L \frac{d^2i}{dt^2} + R \frac{di}{dt} + \frac{i}{C}. \tag{4}$$

The steady-state solutions of these equations give expressions for velocity and current:

$$u = \frac{F}{R_m + j\left(\omega m - \dfrac{1}{\omega C_m}\right)} = \frac{F}{Z_{\text{mech}}} \tag{5}$$

$$i = \frac{V}{R + j\left(\omega L - \dfrac{1}{\omega C}\right)} = \frac{V}{Z_{\text{elect}}}. \tag{6}$$

Now, if the basic analogies between electrical and mechanical quantities are employed [5],

$$i = \alpha u \tag{7}$$

and

$$F = \alpha V, \tag{8}$$

one can obtain the earlier expression for α^2 and subsequently find

$$R + j\left(\omega L - \frac{1}{\omega C}\right) = \frac{R_m}{\alpha^2} + j\left(\frac{\omega m}{\alpha^2} - \frac{1}{\alpha^2 C_m}\right), \tag{9}$$

which leads to the notations of Fig. 1. To simplify further, since the transducer is operated at or near the resonant frequency,

$$\omega_r = \frac{1}{\sqrt{mC_m}} = \frac{1}{\sqrt{LC}}, \tag{10}$$

the reactances of the mechanical arm will cancel, leaving just R. In addition, R will be very much smaller than R_D, and the resulting simplified circuit is shown in Fig. 2.

It is clear from Fig. 2 that the factor of interest is α^2. The next section describes the calculation of the transformation factor from readily available information.

III. A Method for Calculating the Equivalent Circuit

The mechanical energy stored in the transducer is equal to one-half the product of force and extension [3]

$$\omega_m = \tfrac{1}{2} FSl \tag{11}$$

where S is the strain and l is the transducer wall thickness. This can also be written

$$\omega_m = \tfrac{1}{2} F^2 C_m = \tfrac{1}{2} \alpha^2 V^2 C_m = \tfrac{1}{2} V^2 C \tag{12}$$

in which

$$C = \alpha^2 C_m \qquad C_0 = \epsilon A/l$$

as shown by Blitz [6]. The electrical energy supplied is simply

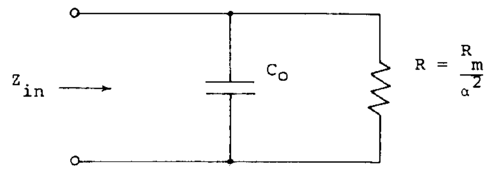

Fig. 2. Simplified equivalent circuit.

$$\omega_e = \tfrac{1}{2} V^2 C_0. \tag{13}$$

The ratio ω_m/ω_e is the electromechanical coupling coefficient k_{31}^2 for the radial vibratory mode. Thus, we can rearrange the above to get

$$\frac{\omega_m}{\omega_e} = \frac{C}{C_0} = \frac{\alpha^2 C_m}{C_0} = k_{31}^2, \tag{14}$$

giving

$$\alpha^2 = \frac{k_{31} C_0}{C_m}. \tag{15}$$

With this equation for α^2, the calculation of C_0 and R for Fig. 2 is straightforward and would proceed as follows. From the manufacturer's data sheet for the material of the transducer we have

frequency constant (used in selecting size) N
dimensions
free relative dielectric constant ϵ_r
density ρ
electromechanical coupling coefficient k_{31}.

The electrical storage element can be found from the equation for a capacitance formed by coaxial cylinders of length l

$$C_0 = \frac{2\pi\epsilon l}{\ln\left(\dfrac{r_2}{r_1}\right)} \text{ F} \tag{16}$$

where r_1 is the inner radius, and r_2 is the outer radius. The resonant frequency of the cylinder is given by

$$\omega_r = \frac{2\pi N}{D} \text{ rad/s} \tag{17}$$

where D is the mean diameter. Next, the mass of the cylinder is calculated from density and volume as

$$m = \rho V = \rho \pi l (r_2^2 - r_1^2) \text{ kg}. \tag{18}$$

The mechanical compliance, from (10), is

$$C_m = \frac{1}{\omega_r^2 m} \tag{19}$$

and the transformation ratio follows from (15). R_m is then found from (1), where $A = \pi Dl$, and R (Fig. 2) is just R_m/α^2.

IV. Results

These calculations were performed for transducers of two different lead zirconate/lead titanate compositions.

TABLE I
MATERIAL CONSTANTS

Type	Frequency Constant (kHz · in)	ϵ_r	ρ (kg/m^3)	k_{31}
4	41	1250	7.6×10^3	0.30
5	34	1700	7.8×10^3	0.32

TABLE II
CALCULATED AND MEASURED EQUIVALENT CIRCUIT VALUES

Transducer #	OD (mm)	l (mm)	Wall Thickness (mm)	Material	R(meas) (kΩ)	R(calc) (kΩ)	C_0(meas) (nF)	C_0(calc) (nF)
1	6.3	6.3	0.786	5	1.68	1.64	2.86	3.44
2	12.7	12.7	1.57	5	1.55	1.60	8.80	8.12
3	28.5	12.7	3.17	5	3.64	3.67	6.30	6.23
4	76.0	51.0	5.10	4	2.49	2.61	24.7	24.6

TABLE III
76 MM TRANSDUCER CALCULATIONS

C_0 (nF)	ω_r (rad/s)	M (kg)	C_m (pF)	α^2	R_m (kΩ)	R (kΩ)
24.6	8.59×10^4	0.44	308	7.19	18.8	2.61

Table I shows material constants which are typical of the type "4" and "5" compositions used. While the "4" material has a lower power factor and is more suited to transmitting applications, the "5" material is preferred for moderate and low power levels due to its stability, high dielectric constant and high coupling coefficient.

The mounted transducers were subsequently measured in a test tank using a low frequency impedance bridge and an ultrasonic receiver as a detector. The inside surface of all transducers was acoustically isolated by lining with a layer of cork that was then coated with rubberized adhesive. The entire unit was then encapsulated in epoxy resin to produce a tough waterproof unit. The thickness of the epoxy coating on the outside face was approximately equal to the transducer wall thickness in all cases. The only end clamping effect is then due to the bobbin-shaped mass of epoxy from end to end. The results of calculations and measurements for four typical transducers are given in Table II.

As can be seen, there is excellent agreement between measured and calculated values of R. If it is assumed that the effect of the epoxy mounting is to both clamp the transducer (decreasing compliance) and increase the vibrating mass, ω_r will remain constant if these effects are about equal. However, α^2 will increase, and a small decrease in R would be expected. This effect is observed in the larger transducers. The largest errors occurred in the calculated values of C_0 for the two smallest transducers. This can be attributed to the fact that the tolerances on physical dimensions become a larger percentage of the overall dimensions as the outside diameter (OD) decreases. Consequently, a small error in measuring the wall thickness can lead to a large error in the value of C_0 (calc). For this reason it may be preferable, if possible, to obtain C_0 by a low frequency bridge measurement on the unmounted or mounted transducer.

V. APPLICATION

We have used the procedure described in many applications. One example is an oceanographic telemeter for conductivity, temperature, and depth [7]. The device was designed for attachment to towed plankton sampling nets. A microcomputer converts the sensor readings and encodes them for pulse position modulation (PPM) transmission to a hydrophone on the towing vessel. The 76 mm OD transducer of Table II was used to deliver 10 W acoustic power to the water.

The output transducer was characterized as described earlier. C_0 and ω_r are determined from (16) and (17), mass and compliance from (18) and (19), and α^2 follows from (15). Results of these calculations are shown in Table III.

In transmitting applications, the usual practice is to tune out C_0 with an inductance to present a resistive load to a narrow-band power amplifier. Since efficiency is of paramount importance in battery operated telemeters, a switched-mode Class D or E amplifier is most appropriate [8]. Accordingly, a Class D amplifier was designed to deliver 12.5 W (overall electric-to-acoustic efficiency assumed to be 80 percent). The voltage-switching configuration was used with a supply voltage of 24 V. The required load resistance is then [9]

$$R_L = \frac{2(V_{cc} - 2V_{\text{sat}})^2}{\pi^2 P_{\text{out}}} = 9.03\ \Omega. \tag{20}$$

Since R_L is less than R or its series equivalent, a trans-

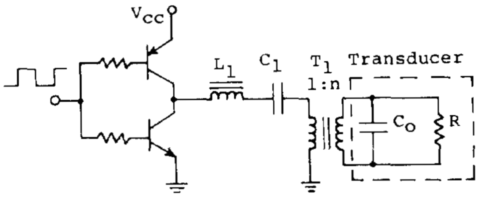

Fig. 3. Voltage-switching Class D amplifier.

former is used to match R to R_L and to resonate C_0. The resulting circuit is shown in Fig. 3.

The loaded Q for the series circuit is selected to be five, giving

$$L_1 = 524\ \mu\text{H}$$

and

$$C_1 = 259\ \text{nF}.$$

The required turns ratio n and the inductance of $T1$ secondary are

$$n = \sqrt{R/R_L} = 17 \tag{21}$$

$$L = 1/\omega_r^2 C_0 = 5.5\ \text{mH}. \tag{22}$$

The amplifier/transducer combination of Fig. 3 has been used in similar applications with ranges of up to 10 km in the open ocean.

VI. Conclusion

A procedure for calculating the simplified equivalent circuit of a radially vibrating ceramic cylinder has been described. While the number and type of transducers tested and characterized was necessarily limited, the method could be extended to other shapes and other modes of vibration when proper coupling coefficients and mounting conditions are considered. The calculated results are sufficiently accurate to determine tuning and power amplifier requirements for a given application.

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