

pass through a solid symbol should be broken away to leave a space around outside the symbol.

The width of a curve should be about  $H/160$ ; if there are several curves,  $H/200$ . The width of lines, bounding a grid may be  $H/250$ . The width of other grid lines may be  $H/500$ . For more details see "American National Standard Illustrations for Publication and Projection," Y15.1-1959. See also *American Institute of Physics Style Manual*, revised 1973.

Although it is possible to put a table of 20 lines of readable numbers on a slide, the pile of numbers is too great for ready comprehension. Try to limit the number of lines in a table to six; the fewer the better.

Many acoustical graphs depict a level of some kind versus the logarithm of frequency ratio. The scale

proportions for such graphs are specified in International Electrotechnical Commission Publication 263-1975. The length for the 10:1 frequency ratio is to equal that for 25 decibels, or 50 decibels or 10 decibels. Intercomparison of original graphs is facilitated by regular use of 2 mm for 1 dB.

Figure 1 is an example of a graph prepared in consonance with the several guidelines described above. It is here reproduced  $\frac{1}{2}$  original size.

The legend printed underneath Fig. 1 is intended to answer the universal questions: *who, what, when, where*. To make it meaningful, a measurement of sound pressure level must always be accompanied by *where*, as by saying "50 ft from the centerline of the passing truck."

## Difficulties of choosing a frequency for miniature transmitters in fresh water

Douglas Pincock

*Department of Electrical Engineering, University of New Brunswick, Fredericton, New Brunswick, Canada*  
(Received 20 December 1976)

It is shown that the choice of a suitable operating frequency for miniature acoustic transmitters depends crucially on a knowledge of absorption and noise characteristics of the water. For application in fresh water, imperfect knowledge of these characteristics can result in incorrect frequency choice giving serious reductions in range capability for the transmitting power available.

PACS numbers: 43.30.Vh, 43.30.Yj

For some time at the University of New Brunswick, we have been engaged in the development of miniature battery-powered acoustic transmitters either as simple locator beacons or else for the transmission of information from a sensor. The principal application has been studies involving fish behavior in which transmitters are attached to the fish to be studied. Naturally, minimization of the size of the transmitter is of prime importance.

The size of two major components of the transmitter—the battery and the electric-to-acoustic transducer—are strongly affected by the choice of transmission frequency. First, the acoustic energy, and hence battery energy, required to achieve a particular range is frequency dependent as a result of variations of both absorption and noise level with frequency. Secondly, the size of the resonant dimension of the transducer is inversely proportional to frequency. Thus, the choice of frequency usually involves finding the middle ground between the small transducer but large acoustic output required at high frequencies and the large transducer but smaller acoustic output at low ones.

This frequency choice can be made rather easily if, for a given range requirement and a set of receiver characteristics, the dependence of acoustic output necessary on frequency is known. For the ocean, where absorption and noise are well characterized, such a dependence is easily determined<sup>1</sup> and we have successfully

designed transmitters for various range requirements between 200 and 5000 m with transmission frequencies ranging from 20 to 100 kHz. In no case were there any significant discrepancies between predicted and measured ranges.

In fresh water, however, predictions of range can only be approximate because of the lack of comprehensive data on either absorption or noise. In particular, absorption will vary considerably from one body of water to another because of varying amounts of suspended matter which seem to exist in even very clean water.<sup>2</sup> Also, there is no reason to believe that low-frequency noise in lakes and rivers will be identical to that in the ocean. Nevertheless, on several occasions no significant errors have resulted from the assumptions that noise conditions are the same as in the sea and absorption as for distilled water. Recent experience, though, has shown that such assumptions are not always satisfactory.

In order to achieve a very small transmitter for salmon smolt studies, a frequency of 150 kHz was chosen permitting the realization of a transmitter with a useful life of several weeks in a cylindrical package  $6 \times 25$  mm<sup>2</sup>. Using the assumptions described above, a range in fresh water of about 300 m (i. e., 50 dB of attenuation between transmitter and receiver) was predicted. Preliminary work in a clean lake seemed to confirm this prediction, but during subsequent tests in the area

of interest (the Penobscot River of Maine during spring runoff) a range capability of only 50 m or less was achieved. Such a result indicates a difference of approximately 15 dB between either the predicted and actual absorption or the predicted and actual noise levels.

The fact that little degradation was noted in the performance of a 50-kHz transmitter of comparable acoustic output rules out an explanation based on unforeseen noise levels. On the other hand, the lack of degradation at 50 kHz is perfectly consistent with absorption due to suspended materials since the logarithmic coefficient of such absorption is proportional to frequency squared.<sup>3</sup> Since water samples were not taken, the nature of any particles present is not known; but, because of the season, silt carried by the river was probably the dominant effect.

The above experience indicates then, that, because of the extreme variations, particularly in absorption

conditions, which exist from one body of fresh water to another, the choice of an operating frequency for a particular application is difficult. This situation can be largely remedied by the measurement of absorption conditions in a large number of typical locations. Of course, such measurements should be correlated with an analysis of water impurities present (suspended particles, air bubbles, etc.) and compared to theoretical predictions based on such analysis. In addition, measurements of noise spectra would be useful.

<sup>1</sup>D. G. Pincock and D. McG. Luke, "Systems for Telemetry from Free Swimming Fish," in Proceedings of the Conference on Instrumentation in Oceanography, 23-25 September 1975, Bangor, Wales (unpublished).  
<sup>2</sup>H. Braithwaite, "Some Measurements of Acoustic Conditions in Rivers," J. Sound Vib. 37, 557-563 (1974).  
<sup>3</sup>R. Meister and R. St. Laurent, "Ultrasonic Absorption and Velocity in Water Containing Algae in Suspension," J. Acoust. Soc. Am. 32, 556-559 (1960).

## Comments on "Wave propagation in a viscoelastic rod with temperature-dependent properties" [E. C. Ting, J. Acoust. Soc. Am. 58, 1018-1022 (1975)]

Y. Vasudeva Rao and K. Bhaskara Rao

Department of Applied Mathematics, Andhra University, Waltair, India  
 (Received 12 November 1976)

Ting's solution for the problem of wave propagation in a viscoelastic rod with temperature-dependent properties is shown to be obtainable due to a more straightforward method due to Keller and Keller.

PACS numbers: 43.40.Cw, 43.20.Bi, 43.35.Mr

Extending the work of Lee and Kanter,<sup>1</sup> Ting<sup>2</sup> studied wave propagation in a viscoelastic rod of the Maxwell type whose mechanical properties were assumed to be temperature dependent.

The partial differential equation governing the motion of a semi-infinite linear viscoelastic rod of Maxwell type—neglecting the inertia effect of the associated lateral motion—together with the associated boundary conditions is transformed into a linear second-order ordinary differential equation and again converted into a nonlinear first-order ordinary differential equation by means of a simple transformation. For a solution a perturbation technique in which higher-order terms are neglected is developed.

The note is concerned with obtaining the same result through a more analytical method, due to Keller and Keller,<sup>3</sup> treating the problem at the ordinary differential equation stage.

The transformed problem<sup>2</sup> is

$$\frac{d^2 \bar{\sigma}}{dx^2} - \frac{1}{c^2(x)} \left( s^2 + \frac{s}{\tau(x)} \right) \bar{\sigma} = 0, \tag{1}$$

$$\frac{d\bar{\sigma}}{dx} = \rho s \bar{u}, \tag{2}$$

$$\bar{\sigma} = 0 \text{ as } x \rightarrow \infty, \quad \bar{u} = V/s \text{ at } x = 0. \tag{3}$$

Combining (1) and (2), we get

$$\frac{dU}{dx} = A(x)U, \tag{4}$$

where  $U = (\bar{\sigma}, \bar{\sigma}')^T$  and

$$A(x) = \begin{pmatrix} 0 & 1 \\ [1/c^2(x)][s^2 + s/\tau(x)] & 0 \end{pmatrix}.$$

Let us write

$$\bar{\sigma}(0) = a \text{ and } \bar{\sigma}'(0) = b = \rho V. \tag{5}$$

For (4) and (5), Keller's exponential-like solution—the convergent criterion for which is relatively less stringent—after neglecting the integral coefficients, is

$$\bar{\sigma} = \frac{D\sqrt{c}}{(s^2 + s/\tau)^{1/4}} \exp \left[ - \int_0^x \left( s^2 + \frac{s}{\tau(\bar{x})} \right)^{1/2} \frac{d\bar{x}}{c} \right]$$

and the constant  $D$  is determined employing the velocity condition in (5).

It is of analytical interest to check if neglecting the higher-order terms in perturbation together with avoiding the higher-order terms in binomial expansion of